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Modelling the flow conditions in the tunnel and its reduced model †

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Abstract

The article discusses the problems of flow condition modelling on real object as well as its reduced model while using similarity theory. Similarity criteria and model flow laws are derived by means of dimensional analysis. Presented is the computation of flow through the tunnel and its reduced model as well as the applied numerical methods.

Keywords: Dimensional analysis; Real object; Reduced model; Physical similarity of flow

1. Introduction

Two basic problems are analyzed in this article. First one is oriented to evaluation of similarity theory application on fluid flow modelling in the tunnel and its reduced model. The crucial knowledge resulting from this part of the solution is summarized separately. The other problem is related to the investigation of air compressibility influence on computation results of determining flow parameters on the model.

The theory of physical modelling underlies the investigation of hydrodynamic and aerodynamic phenomena connected with flow similarity examination on real objects and its reduced model. It mostly points out what requirements the scaled physical model must meet in order to simulate most closely real hydrodynamic or aerodynamic phenomena in the reduced scale. It also helps to identify which variables are needed to be measured during the experiment, how to process the results and how to determine the range of their validity.

Besides physical modelling, also mathematical modelling including advanced numerical methods can be applied for fluid flow investigation [1-4].

2. Application of dimensional analysis in the theory of similarity

The theory of similarity in general is elaborated in two ways. The first one is based on determination of similarity criteria by analysis of differential equations of motion which describe examined physical phenomena mathematically, whereas the other way takes *dimensional analysis* as basic access. The first method is suitable for research on models if a relevant mathematical formulation of examined phenomena is available. In case of absence of mathematical description the application of dimensional analysis requires detailed analysis of phenomena essence.

Dimensional analysis is a method of determining variables related to the investigated phenomenon and also a tool for setting the minimal number of non-dimensional arguments composed of them. Basic general principles of dimensional analysis are presented, e.g., in [5-8].

Two cognate methods of dimensional analysis are the most significant for the application in practice:
- Rayleigh's method,
- Buckingham's method (so called π- theorem).

The theory of physical modelling of the flow conditions is based on the *mechanical similarity laws of fluid flow*. They are the laws which rule the relations among properties of a certain system (real object) and properties of reduced model of this system [9]. Actually, it is to find mutual relations among variables of these two systems which we try to formulate mathematically in the form of so-called *model laws of flow*.

3. Modelling the flow conditions on real object and its reduced model

For deriving the mechanical similarity laws of fluid flow in this contribution, Buckingham's method was applied. By means of it the determination process of crucial criterial numbers is described which generally describe fluid flow and also interactions (force effect F) of flow with steady surface in case

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of flow around bodies. Consequently, model laws will be used to model the flow conditions in the tunnel with variable flow cross-section.

From the analysis of flow issue it follows that during examination of flow and flow interaction the following variables will affect the examined phenomena in crucial rate [6, 8, 9]:

- ρ-density (kg·m^{-3}),
- υ-fluid flow velocity (m·s^{-1}),
- ν -kinematic viscosity (m^2·s^{-1}),
- l-characteristic length (m),
- F-force (kg·m·s^{-2}),
- a-speed of sound (m·s^{-1}).

Dependence of presented variables is expressed by the complete physical equation in the form

$$f = (\rho ,\ \upsilon,\ \nu ,\ l,\ F, a) = 0\,. \tag{1}$$

Eq. (1) is dimensionally homogeneous so the variables in it cannot be found separately but in the form of products in the sense of relation

$$\pi = \prod_{i=1}^{n} \varphi_i^{x_i} \ , \tag{2}$$

where π is a non-dimensional variable (similarity criterion, invariant of similarity) (1), n is the number of relevant variables φ, $i \in \langle 1, n \rangle$ and x_i is an exponent (rational number).
In accordance with the Eq. (2), Eq. (1) can be rewritten as

$$\pi = \rho^{x_1} \cdot \upsilon^{x_2} \cdot \nu^{x_3} \cdot l^{x_4} \cdot F^{x_5} \cdot a^{x_6}\,. \tag{3}$$

Among six selected variables we can find two different velocities defined in the same units. The ratio of these velocities (e.g., υ/a) represents a so-called simplex. Therefore, in creating the dimensional matrix in the sense of dimensional analysis requirements, only one from the mentioned velocities will be taken into account. Its selection depends on the problem researcher. In this case the fluid flow velocity υ will be included in the dimensional matrix. This example has five independent variables and three fundamental units. The fundamental units are meter, kilogram, second. Dimensional matrix for basic units of the given magnitudes will consist of $n = 5$ columns and $m = 3$ rows and is defined as

$$\begin{array}{c} \\ \text{kg} \\ \text{m} \\ \text{s} \end{array} \begin{array}{c} \begin{array}{ccccc} \rho & \upsilon & \nu & l & F \end{array} \\ \left\| \begin{array}{ccccc} 1 & 0 & 0 & 0 & 1 \\ -3 & 1 & 2 & 1 & 1 \\ 0 & -1 & -1 & 0 & -2 \end{array} \right\| \end{array} . \tag{4}$$

The number of non-dimensional criteria which result from the solution adds up to $n - m = 2$. The third non-dimensional argument is the above-mentioned simplex which is given by ratio of flow velocity to the speed of sound in this fluid, also known as Mach number (Ma), and is given by

$$Ma = \frac{\upsilon}{a}\,. \tag{5}$$

Based on the value of Mach number all investigated cases of flow around bodies can be divided into two categories:

- subsonic flow ($Ma < 1$),
- supersonic flow ($Ma > 1$).

Considering the influence of fluid compressibility on the calculation result, the relation among fluid flow velocity υ and the speed of sound a in it is essential. If $\upsilon \ll a$, the influence of Mach number is irrelevant, and further it does not have to be considered. The influence of fluid compressibility on the flow conditions is considered at $Ma > 0.2$ [10, 11].

Dimensional matrix (4) will be modified at determining non-dimensional criteria into square matrix

$$\left\| \begin{array}{ccc} 1 & 0 & 0 \\ -3 & 1 & 1 \\ 0 & -1 & 0 \end{array} \right\| \cdot \left\| \begin{array}{c} x_1 \\ x_2 \\ x_4 \end{array} \right\| = (-1) \cdot \left\| \begin{array}{cc} 0 & 1 \\ 2 & 1 \\ -1 & -2 \end{array} \right\| \cdot \left\| \begin{array}{c} x_3 \\ x_5 \end{array} \right\| . \tag{6}$$

Its determinant is $\Delta = 1$. For matrix elements can be derived

$$\begin{array}{rrrcrr} x_1 & & & = & -x_5 & \\ -3x_1 & +x_2 & +x_4 & = & -2x_3 & -x_5 \\ & & -x_2 & = & x_3 & +2x_5 \end{array} .$$

Similarity criteria will be determined in such manner that the indicated system of linear equations will be solved two times, i.e., two independent solutions will be found. For every criterion calculation must be chosen "excessive" unknowns in the equations so that the solution will be unambiguous. The procedure consists in putting one unknown equal to one and the other equal to zero. Selected excessive unknowns must not depend on each other. For this case of flow the selection of unknowns in accordance to the matrix (7) is used:

$$\begin{array}{r} \\ 1.row \\ 2.row \end{array} \begin{array}{c} \begin{array}{cc} x_3 & x_5 \end{array} \\ \left\| \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right\| \end{array} . \tag{7}$$

Two independent vectors result from the solution defined as

$$\begin{array}{c} \\ \pi_1 \\ \pi_2 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \left\| \begin{array}{ccccc} 0 & -1 & 1 & -1 & 0 \\ -1 & -2 & 0 & -2 & 1 \end{array} \right\| \end{array} ,$$

to which two complex non-dimensional arguments correspond in the form of

$$\pi_1 = \frac{\nu}{\upsilon \cdot l} \ , \tag{8}$$

$$\pi_2 = \frac{F}{\rho \cdot \upsilon^2 \cdot l^2}\,. \tag{9}$$

The reciprocal of non-dimensional argument π_1 which is familiar Reynolds number is usually used in the practice [8]

$$Re = \frac{\upsilon \cdot l}{\nu} . \tag{10}$$

In the relation (9) the ratio F/l^2 represents the pressure p so that the non-dimensional argument π_2 is actually the Euler number

$$Eu = \frac{p}{\rho \cdot \upsilon^2} . \tag{11}$$

The third non-dimensional argument π_3 is the already mentioned Mach number (Eq. (5)).

From the above-mentioned it results that there are four dominant forces acting on the body placed in the fluid flow: inertial, frictional, compressive and deformation force. The ratios of these forces are included in three criteria. These are the Reynolds number *Re* (inertial and frictional forces), Euler number *Eu* (compressive and inertial forces), the Mach number *Ma* (inertial and deformation forces).

Non-dimensional form of the function describing the effect of flow on the body in the flow field will be in general defined as

$$f\,(Re, Eu, Ma) = 0 . \tag{12}$$

The force effect of flowing fluid in the non-dimensional description can be expressed from Eq. (12) by the function

$$Eu = f\,(Re,\, Ma) . \tag{13}$$

On assumption that fluid compressibility can be neglected, so if $\upsilon << a$ is valid, the influence of Mach number is not considered and Eq. (13) can be stated

$$Eu = f\,(Re) , \tag{14}$$

or in the detailed form as follows:

$$\frac{p}{\rho \cdot \upsilon^2} \approx \frac{F}{\rho \cdot \upsilon^2 \cdot l^2} = f\left(\frac{\upsilon \cdot l}{\nu}\right). \tag{15}$$

The presented procedure of expressing the basic criterial numbers (non-dimensional arguments, similarity invariants, similarity criteria) confirms that unambiguously for wide area of phenomena in engineering field it is possible to obtain criterial numbers with application of dimensional analysis.

4. Physical similarity, proportionality constants, model laws

Experimental determination of *Eu* dependence on *Re* in the sense of the Eq. (14) is accomplished by placing a body with characteristic dimension of l into the flowing fluid of velocity υ, viscosity ν and density ρ. At these conditions the force (pressure) acting on the body can be measured and the criterial numbers are calculated, from which the dependence of $Eu = f\,(Re)$ can be derived [12]. This relationship can be described in the form of power function. After the measurement of individual physical variables, calculation of the particular values of criterial numbers and their projection in the logarithmic coordinates, we will obtain the dispersion field of points. The regression curve is a straight line for which the regression coefficient (guideline) can be determined as well as the locating constant (distance of intersection point of regression line from zero on the *Eu* coordinate). To every point lying on the obtained line one possible state of fluid flow effect on the body will correspond in which *Eu* and *Re* have constant values. This state in fact represents an infinite number of physically similar cases in which for the reduced model (variables without apostrophe) and the real object (variables marked by apostrophe) will be valid:

$$Eu = Eu' \quad and \quad Re = Re' . \tag{16}$$

In the detailed form

$$\frac{p}{\rho \cdot \upsilon^2} \approx \frac{F}{\rho \cdot \upsilon^2 \cdot l^2} = \frac{p'}{\rho' \cdot \upsilon'^2} \approx \frac{F'}{\rho' \cdot \upsilon'^2 \cdot l'^2} ,$$

and

$$\frac{\upsilon \cdot l}{\nu} = \frac{\upsilon' \cdot l'}{\nu'} . \tag{17}$$

Proportionality constants (scales of change) of particular physical variables on the real object and the model can be defined as

$$\frac{F'}{F} = c_F , \tag{18}$$

$$\frac{\rho'}{\rho} = c_\rho , \tag{19}$$

$$\frac{\upsilon'}{\upsilon} = c_\upsilon , \tag{20}$$

$$\frac{\nu'}{\nu} = c_\nu , \tag{21}$$

$$\frac{l'}{l} = c_l . \tag{22}$$

Taking into account the proportionality constants of individual physical variables defined above and substituting them in relations defined by (17) we can obtain two model laws in the form of

$$1 = \frac{c_p}{c_\rho \cdot c_\upsilon^2} = \frac{c_F}{c_\rho \cdot c_l^2 \cdot c_\upsilon^2} , \tag{23}$$

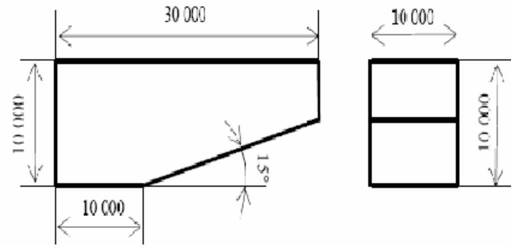

Fig. 1. Basic tunnel characteristics.

$$1 = \frac{c_v \cdot c_l}{c_v} . \qquad (24)$$

In Eqs. (23) and (24) there are in total five proportionality constants. During their application in this case three arbitrary constants are needed to be chosen and the two remaining constants will be calculated from the derived model laws.

5. Flow analysis in the tunnel

The subject of flow condition examination in this contribution is the tunnel with variable flow cross-section which represents the real object. Width of the tunnel and its height up to profile change is 10 m (Fig. 1).

Total length of tunnel is 30 m. Profile change of tunnel occurs after 10 m from its beginning from where the bottom wall of the tunnel is led with declination of 15 degrees up to its outlet. The flowing medium is air with physical properties corresponding to the temperature of 27 °C. Air density is $\rho = 1.24$ kg·m^{-3}, its kinematic viscosity is $\nu = 18 \cdot 10^{-6}$ m^2·s^{-1}, specific heat capacity is $c_p = 1004.4$ J·kg^{-1}·K^{-1}. The inlet velocity is 10 m·s^{-1}.

The solution will be realized by analytical methods and numerical simulation [13]. As the modelled case is quite simple, a relatively "thin" mesh was applied for the numerical solution (Fig. 2).

Software ANSYS_CFX was used for computation, which offers the possibility of mesh refinement and adaptation in the regions where more accurate results are required (e.g., in the boundary layer and so on).

During examination of air flow through the tunnel in the case that air is considered as incompressible, the analytical calculation of flow parameters is based on relation (5). The speed of sound can be determined from equation

$$a = \sqrt{\kappa \cdot r \cdot T_1} , \qquad (25)$$

where κ is adiabatic exponent (for air = 1.4) (1), r is specific gas constant of air (J·kg^{-1}·K^{-1}) and T_1 is air temperature (K).

For specific gas constant of air $r = 287.062$ J·kg^{-1}·K^{-1}, temperature of air $T_1 = 300$ K and adiabatic exponent $\kappa = 1.4$ the calculated speed of sound will correspond to $a = 347$ m·s^{-1} and particular Mach number is

$$Ma' = \frac{\upsilon'}{a'} \cong \frac{10}{347} = 0.0288 . \qquad (26)$$

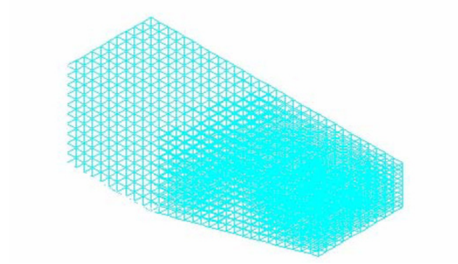

Fig. 2. Computational mesh of the tunnel.

Based on this we can claim that influence of air compressibility during examination of flow conditions in the tunnel is not necessary to be taken into the account.

From analytical solution (continuity equation) it results that the outlet velocity of tunnel in the sense of the relation is

$$\upsilon_2' = \upsilon_l' \cdot \frac{A_1'}{A_2'} = 10 \cdot \frac{100}{46.41} = 21.55 \text{ m} \cdot \text{s}^{-1} , \qquad (27)$$

where υ_l' is the inlet air velocity (10 m·s^{-1}), A_1' is the inlet cross-sectional area of tunnel (100 m^2) and A_2' means the outlet cross-sectional area of tunnel (46.41 m^2).

The Mach number value changes along the length of tunnel, which corresponds to the air velocity in every cross-section of the tunnel. Mach number based on the outlet velocity of 21.55 m·s^{-1} will be

$$Ma' = \frac{\upsilon'}{a'} \cong \frac{21.55}{347} = 0.0621 .$$

Total change of Mach number through the tunnel is in interval $0.0288 < Ma' < 0.0621$. As the calculated Mach number in the whole range of tunnel length is lower than 0.2, for the air flow investigated on the real object condition of its incompressibility can be accepted. This conclusion allows applying the relations (14) and (15) as they were derived from dimensional analysis.

Prediction of the tunnel outlet velocity υ_2' which was obtained by numerical solution with ANSYS_CFX and based on the assumption of incompressible fluid is shown on Fig. 3. Local maximum value of velocity is 21.81 m·s^{-1}. Average value of outlet velocity is 21.75 m·s^{-1}. The error in expressing the outlet velocity by numerical methods in comparison with the analytical methods is 0.93 %.

In case air is considered compressible, the average outlet velocity of the tunnel is 21.79 m·s^{-1} (Fig. 4). Maximal local velocity is 21.85 m·s^{-1}. The inaccuracy in expressing the outlet velocity of compressible medium in comparison to the analytical way is 1.11 %. Compressibility of air was taken into account through the equation of state of ideal gas.

Based on the results of tunnel outlet velocity calculation, it can be concluded that both analytical and numerical methods can be applied to determine the flow parameters no matter if compressibility is taken into account or not. Numerical solu-

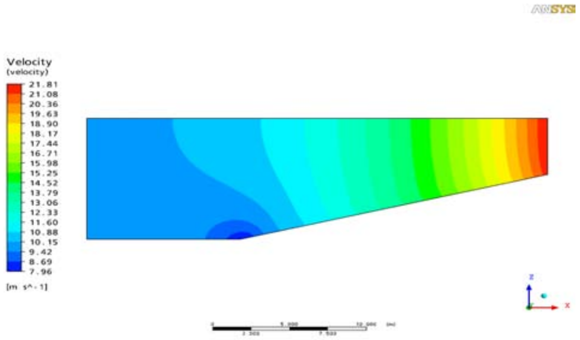

Fig. 3. Velocity contours of incompressible fluid in real tunnel.

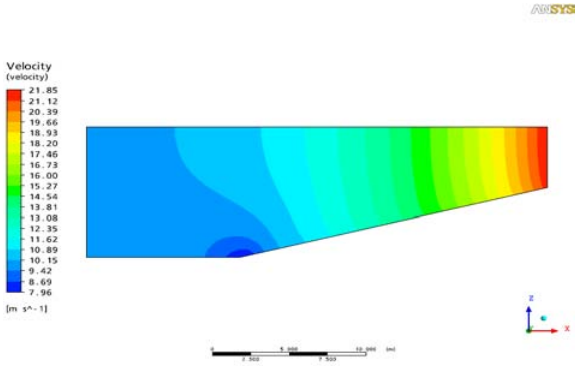

Fig. 4. Velocity contours of compressible fluid in real tunnel.

tion enables one to obtain the velocity distribution along the tunnel cross-section, which in case of analytical methods application is not available [14].

6. Application of model laws on the scaled model of the tunnel

If flow conditions are examined on a reduced tunnel (model), knowledge obtained from dimensional analysis presented by model laws (23) and (24) will be applied. To create the model, we start with the choice of length proportionality constant. This constant will be selected as $c_l = 10$ for the model. Thereby it is expected that all geometric parameters are reduced in comparison to the real object 10 times. From Eq. (22) it results that $l = l'/10$. It also concerns the generated mesh for the flow parameters computation by numerical methods.

If we assume the same fluid in the model as in the case of a real tunnel, the proportionality constant of viscosity and density must also be the same and for them it is valid $c_v = c_\rho = 1$. From Eq. (24) it results that the inlet air velocity on model must be 10-times higher than that of the real object, i.e., its value is 100 m·s^{-1}. It is valid.

$$c_\upsilon = \frac{c_v}{c_l} = \frac{1}{10} \quad \Rightarrow \quad \frac{\upsilon'}{\upsilon} = 0.1 \quad \Rightarrow \quad \upsilon = 10 \cdot \upsilon' .$$

From relation (23) we can express the proportionality constant of force effect on a surface situated in the flow field:

$$c_F = c_\rho \cdot c_l^2 \cdot c_\upsilon^2 \quad \Rightarrow \quad c_F = 1 \cdot 10^2 \cdot \frac{1}{10^2} = 1 .$$

From the proportionality constant of force (18) it results that the compression force acting on the object situated in the flow field in the real tunnel will be the same as the force acting on the object situated in the flow field in model, whereas it is valid:

$$c_F \cdot F = F' \quad \Rightarrow \quad F = F' .$$

In the case of air as incompressible fluid we can derive from analytical solution that the value of Mach number in the inlet cross-section is $Ma' = 0.0288$ and in the outlet $Ma' = 0.0621$. At the same conditions in the tunnel scaled model and inlet velocity $\upsilon_1 = 100$ m.s^{-1} the value of Mach number is $Ma = 0.288$. Using Eq. (27), the determined outlet velocity of the model is $\upsilon_2 = 215.5$ m·s^{-1}. Mach number corresponding to this velocity is $Ma = 0.621$.

Since the Mach number at the inlet and outlet cross-section of the model obtained by analytical method is $Ma > 0.2$, it is necessary to consider the influence of air compressibility during investigation of flow conditions. In the situation when air is applied to model the flow both in the real tunnel and its reduced model, it is impossible to claim neither agreement of physical essence of flow nor agreement of mathematical description of phenomena investigated on the real tunnel and its model. This knowledge results from calculated values of the Mach number. Taking air compressibility into account, the value of the Mach number for the model can only be found by numerical simulation. This value fundamentally differs from that calculated in the case of incompressible air.

If the inlet air velocity in the model reaches 100 m·s^{-1}, the flow in the outlet cross-section can occur in critical conditions. Outlet velocity cannot be supercritical for this shape of cross section and the value of Mach number must be $Ma \le 1$. The state at which the critical velocity υ^* is reached in outlet cross-section is marked as *aerodynamic stall.* In these conditions it is possible to increase the mass flux only by change of static flow parameters, such as temperature. In problems connected with high flow velocities it is always necessary to realize analysis oriented to the control of critical flow parameters. In this particular case of air flow through scaled model of tunnel with inlet velocity of 100 m·s^{-1}, the analytical method will be applied to judge the possibility of aerodynamic stall occurrence.

To control the given inlet velocity condition, the relation characterizing mass flux at stationary 1D flow of compressible fluid will be applied [1]:

$$\frac{\rho_1 \cdot \upsilon_1}{\rho^* \cdot \upsilon^*} = \left(\frac{\kappa + 1}{2} \right)^{\frac{1}{\kappa - 1}} \cdot M^* \cdot \left(1 - \frac{\kappa - 1}{\kappa + 1} \cdot M^{*2} \right)^{\frac{1}{\kappa - 1}} , \qquad (28)$$

where υ_1 is inlet velocity of fluid (m·s^{-1}), ρ_1 is inlet density of fluid (kg·m^{-3}), υ^* is critical velocity (m·s^{-1}), ρ^* is fluid density in the critical cross-section of model (kg·m^{-3}).

Non-dimensional velocity M^* is determined as ratio of velocity υ and the speed of sound at critical parameters a^*

$$M^* = \frac{\upsilon}{a^*} .$$

The left side of Eq. (28) represents the ratio of areas in the critical and inlet cross-sections derived from the continuity equation,

$$\frac{\rho_1 \cdot \upsilon_1}{\rho^* \cdot \upsilon^*} = \frac{A}{A_1} \quad \Rightarrow \quad \frac{A^*}{A_1} = \frac{A_2}{A_1} = 0.4641 , \qquad (29)$$

where A_1 is the inlet area of model and $A_2 = A^*$ is outlet area of model at 15° inclination of its bottom part.

Assuming M^*=1 then outlet area of the model $A_2 = A^*$. Air velocity in the cross-section reaches the value of critical velocity $\upsilon^* = a^*$. This velocity is given by relation (25) where for temperature T_1 will be substituted T^* given by [15, 16].

$$T^* = \frac{2}{\kappa + 1} \cdot T_1 ,$$

and the calculated critical velocity is $a^* = 317$ m.s^{-1}.

Maximum inlet velocity at which it will not come to the stall is determined from the relation

$$\upsilon_{1\max} = M^* \cdot \upsilon^* = M^* \cdot a , \qquad (30)$$

where the non-dimensional velocity M^* in the critical cross-section is determined from relation (28). The process proceeds as follows: for the ratio of areas $A^*/A_1 = 0.4641$ in the sense of relation (29), Eq. (28) is defined in the form

$$\left(\frac{1.4+1}{2}\right)^{\frac{1}{1.4-1}} \cdot M^* \cdot \left(1 - \frac{1.4-1}{1.4+1} \cdot M^{*2}\right)^{\frac{1}{1.4-1}} - 0.4641 = 0 . \qquad (31)$$

By solving Eq. (31) we can obtain two roots. A graph of the function on the left hand side of the equation shows that in area of *subsonic* flow non-dimensional velocity M^* has one solution and in the area of *supersonic* flow also one solution. If Eq. (31) is solved numerically, (e.g., bisection), the computed value of non-dimensional velocity is $M^* = 0.3060$ for the area of subsonic flow and $M^* = 1.7490$ for area of supersonic flow (Fig. 5).

In the area of subsonic flow, the maximum possible inlet air velocity of model at which it will not come to aerodynamic stall of tunnel is determined from (30):

$$\upsilon_{1,\max} = M^* \cdot a^* .$$

Its value is $\upsilon_{1,\max} = 0.3060 \cdot 317 = 97$ m·s^{-1}.

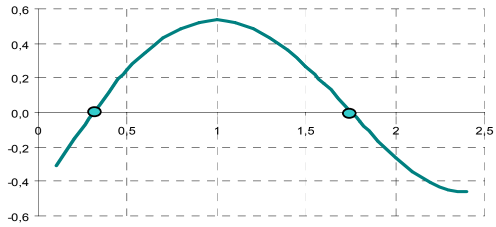

Fig. 5. Equation roots (28) in area of subsonic and supersonic flow.

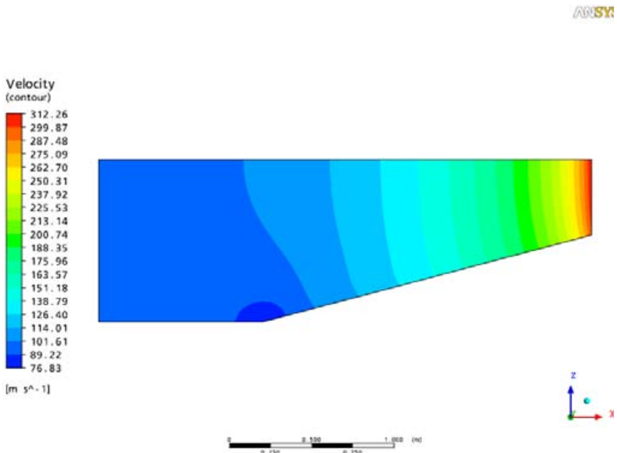

Fig. 6. Cross-sectional velocity distribution on scaled model of the tunnel.

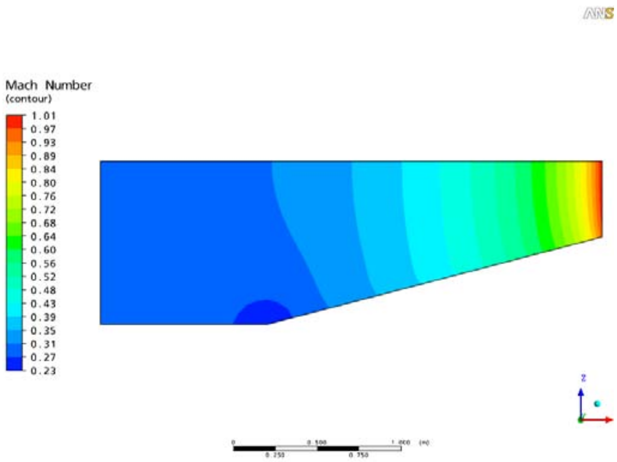

Fig. 7. Cross-sectional Mach No. distribution on scaled model of the tunnel.

Numerical solution enables one to obtain the information about other parameters representing compressible air flow through the model. Fig. 6 shows the velocity distribution along the model length. Maximal outlet velocity is 312.26 m·s^{-1} and the mean outlet velocity is 308.7 m·s^{-1}. The Mach number in the outlet cross-section reaches a mean value of 0.9951 (Fig. 7). In Fig. 8 the variation of air temperature along the model length corresponding to the character of flow is plotted. Mean temperature is 27 °C in the inlet cross-section and in outlet cross-section is air cooled to minus 16.86 °C.

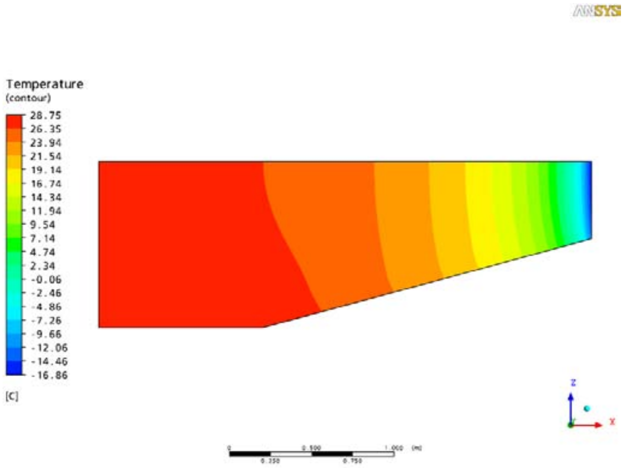

Fig. 8. Cross-sectional temperature distribution on scaled model of the tunnel.

7. Conclusion

From analysis of Mach number computation on the scaled model and real object the result is that modelling the flow conditions on a model in scale 1:10 by means of air is not possible. Compressibility of air can be neglected in case of a real tunnel but not on the model.

To avoid difficulties in physical modelling of the flow resulting from the condition of respecting the Reynolds number, the usage of different fluid on the model is required.

Using the same fluid both on the real object and the scaled model leads to significantly higher velocities on the model. This results from the relation between proportionality constant of velocity and proportionality constant of length $c_\upsilon = c_l^{-1}$.

In case of modelling the flow conditions on the scaled tunnel (model) by means of water, it is necessary to consider its kinematic viscosity at 27 °C, which is $0.9.10^{-6}$ $m^2 \cdot s^{-1}$. The kinematic viscosity of air at the same temperature is 18.10^{-6} $m^2 \cdot s^{-1}$ and density 1.24 $kg \cdot m^{-3}$. The proportionality constant of viscosity in accordance with relation (21) will correspond to value $c_\nu = \nu / \nu' = 18 \cdot 10^{-6} / 0.9 \cdot 10^{-6} = 20$ and proportionality constant of density $c_\rho = \rho' / \rho = 1.24/1000 = 0.00124$. From Eq. (20) it results that the proportionality constant of velocity at selected proportionality constant of length $c_l = 10$ is $c_\upsilon = c_\nu / c_l = 20/10 = 2$. Because simultaneously for a proportionality constant of velocity is $c_\upsilon = \upsilon' / \upsilon$, inlet water flow velocity on the model will be $\upsilon = \upsilon' / c_\upsilon = 10/2 = 5$ $m \cdot s^{-1}$ (in case of air it was $\upsilon = 100$ $m \cdot s^{-1}$). Water velocity in outlet cross-section will in sense of relation (27) respond to value 10.84 $m \cdot s^{-1}$.

From the above-mentioned values of particular proportionality constants and from model law (23) results a proportionality constant of force $c_F = c_\rho \cdot c_\nu^2 = c_\rho \cdot c_\upsilon^2 \cdot c_l^2$ which is $c_F = 0.00124 \cdot 2^2 \cdot 10^2 = 0.496$.

From this analysis it can be concluded that in case of the strength calculation (i.e., calculation of stiffness and stress conditions) on the body situated in the air flow in the real tunnel we can expect the force which will be in result 0.496 times lower than force found out on model, whereas the proportionality constant of force is defined as $F'/F = 0.496$.

In case of using two different fluids at examining flow conditions on model and real object, it is not possible to consider the same force effect on the body situated in the flow field in the tunnel and its model. If we do so, we obtain such proportionality constant of density and viscosity that practically no fluid with corresponding viscosity and density can be found. Because the selection of a suitable fluid is limited in real practice mostly to water and air, it is necessary to realize that the force determined on the model needs to be corrected through a proportionality constant of force c_F during conversion to dimensions of the real tunnel.

As stated above, the modelling of the flow conditions on reduced tunnel model by means of air in scale 1:10 is impossible. At such reduction of tunnel dimensions it is not possible to reach correspondence of physical essence of flow on a real object and its model. If this fact was underestimated, the results from solving the flow conditions on the model would not reflect the real state when transformed to a real object. In case of high velocities reached on the model, it is recommended to use other fluids, e.g., water during the flow investigation. Results obtained by modelling the water flow on the model can consequently be transferred to the real tunnel. The correspondence of physical essence of flow between the real object and its model will be ensured if the method described above will be applied.

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